$\left(x^2+1\right)y''-4xy'+6y=0$ denklemini x=0 adi nokta civarında çözünüz.

$$y(x)=\sum_{n=0}^{\infty}a_nx^n \qquad y'(x)=\sum_{n=1}^{\infty}na_nx^{n-1} \qquad y''(x)=\sum_{n=2}^{\infty}n(n-1)a_nx^{n-2}$$

$$\left(x^2+1\right)\sum_{n=2}^{\infty}n(n-1)a_nx^{n-2}-4x\sum_{n=1}^{\infty}na_nx^{n-1}+6\sum_{n=0}^{\infty}a_nx^n=0$$

$$\sum_{n=2}^{\infty}n(n-1)a_nx^n+\sum_{n=2}^{\infty}n(n-1)a_nx^{n-2}-\sum_{n=1}^{\infty}4na_nx^n+\sum_{n=0}^{\infty}6a_nx^n=0$$

$$\sum_{n=2}^{\infty}n(n-1)a_nx^n+\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n-\sum_{n=1}^{\infty}4na_nx^n+\sum_{n=0}^{\infty}6a_nx^n=0$$

$$\sum_{n=0}^{\infty}\left[n(n-1)a_n+(n+2)(n+1)a_{n+2}-4na_n+6a_n\right]x^n=0$$

$$\sum_{n=0}^{\infty}\left[\left(n^2-5n+6\right)a_n+(n+2)(n+1)a_{n+2}\right]x^n=0$$

$$\sum_{n=0}^{\infty}\left[(n-2)(n-3)a_n+(n+2)(n+1)a_{n+2}\right]x^n=0$$

$$(n-2)(n-3)a_n+(n+2)(n+1)a_{n+2},\quad n=0,1,2,\ldots$$

Rekürans bağıntısı

$$a_{n+2}=-\frac{(n-2)(n-3)a_n}{(n+2)(n+1)},\qquad n=0,1,2,\ldots$$

$n=0$: $\qquad a_2=-3a_0$

$n=1$: $\qquad a_3=-\frac{1}{3}a_1$

$n=2$: $\qquad a_4=-\frac{0}{12}a_2=0$

$n=3$: $\qquad a_5=-\frac{0}{20}a_3=0$

$$y(x)=a_0\left\{1-3x^2\right\}+a_1\left\{x-\frac{1}{3}x^3\right\}$$

$x^2y''+xy'+(x^2-1/16)y=0$ (1)

denklemi için x= 0 noktasının düzgün tekil nokta olduğunu gösterin, İndis denkleminin köklerini bulun ve çözün.

P(x) =x^2=0 $\quad$ x=x_0 =0 tekil nokta için

p_0= $\lim_{x\to 0}(x)\dfrac{Q(x)}{P(x)}$ **=** $\lim_{x\to 0} x\dfrac{x}{x^2}=1$ $\qquad$ **q_0=** $\lim_{x\to 0} x^2\dfrac{x^2-m^2}{x^2}=-1/16$

x_0=0 tekil noktası düzgün tekil noktadır.

F(r)= r(r-1)+p_0r+q_0 denkleminde yerlerine konursa

$r^2-1/16=0$ indis denklemi kökler r=$\pm 1/4$

y= $\sum_{n=0}^{\infty} a_n x^{r+n}$

$y'=\sum_{n=0}^{\infty}(r+n)a_n x^{r+n-1}$

$y''=\sum_{n=0}^{\infty}(r+n)(r+n-1)a_n x^{r+n-2}$ (1) de yerlerine konur ve düzenlenirse

$$\sum_{n=0}^{\infty}(r+n)(r+n-1)a_n x^{r+n}+\sum_{n=0}^{\infty}(r+n)a_n x^{r+n}+\sum_{n=0}^{\infty}a_n x^{r+n+2}-\sum_{n=0}^{\infty}\frac{1}{16}a_n x^{r+n}=0$$

Bu seriyi x^{r+n} parantezine alabilmek için 3. terimde indis ötelemesi yapılırsa (n=n-2)yazılırsa

$$\sum_{n=0}^{\infty}(r+n)(r+n-1)a_n x^{r+n}+\sum_{n=0}^{\infty}(r+n)a_n x^{r+n}+\sum_{n=2}^{\infty}a_{n-2} x^{r+n}-\sum_{n=0}^{\infty}\frac{1}{16}a_n x^{r+n}=0$$

$$L(\phi)(r,x)=\sum_{n=0}^{\infty}\left[(r+n)(r+n-1)+(r+n)-\frac{1}{16}\right]a_n x^{r+n}+\sum_{n=0}^{\infty}a_n x^{r+n+2}=(r^2-\frac{1}{16})a_0x^r+\left[(r+1)^2-\frac{1}{16}\right]a_1x^{r+1}+$$

$$+\sum_{n=2}^{\infty}\left(\left[(r+n)^2-\frac{1}{16}\right]a_n+a_{n-2}\right)x^{r+n}=0 \qquad (2)$$

x in en küçük üssüne sahip (x^r) nin katsayısı 0 a eşitlenerek indis denklemi elde edilir. ($a_0\neq 0$)

$$r^2-\frac{1}{16}=0$$

İndis denkleminin kökleri (r_1=1/4, ve r_2=-1/4) dir. Rekürans bağıntısı

$((r+n)^2-\frac{1}{16})a_n+a_{n-2})x^{r+n}=0$ $\quad$ n≥2 ile

$$a_n = \frac{-a_{n-2}}{(r+n)^2 - 1/16}. \qquad (3)$$

olur.

(2) denklemindeki x^{r+1} in katsayısından a_1=0 bulunur. Böylece (3) denkleminden **a_3=a_5=..........=$a_{(2n+1)}$=0** olur.

Her köke karşılık gelen bağıntı yazılır. n=2,3,4,...verilerek katsayılar bulunur.

r_1=1/4 r_2=-1/4

$$a_n = \frac{-2a_{n-2}}{2n^2 + n}$$

$$a_n = \frac{-2a_{n-2}}{2n^2 - n}$$

n=2 için a_2= $a_2 = \frac{-2a_0}{10} = -\frac{a_0}{5}$ n=2 için a_2= $a_2 = \frac{-2a_0}{6} = -\frac{a_0}{3}$

n=4 için $a_4 = \frac{-2a_2}{36} = -\frac{a_2}{18} = \frac{a_0}{90}$ n=4 için $a_4 = \frac{-2a_2}{28} = -\frac{a_2}{14} = \frac{a_0}{72}$

Her kök değeri için ayrı **y=** $\sum_{n=0}^{\infty} a_n x^{r+n}$ **=a_0x^r + a_2x^{r+2}+ a_4x^{r+4}** denkleminde yerlerine konarak **çözümler bulunur**

$$y_1(x) = x^{1/4}(a_0 - \frac{a_0}{5}x^2 + \frac{a_0}{90}x^4) = x^{1/4}a_0(1 - \frac{x^2}{5} + \frac{x^4}{90})$$

a_0=1 seçilerek

$$y2(x) = x^{-1/4}(a_0 - \frac{a_0}{3}x^2 + \frac{a_0}{72}x^4) = x^{-1/4}a_0(1 - \frac{x^2}{3} + \frac{x^4}{72})$$

y_{genel}=$c_1y_1(x)$+$c_2y_2(x)$ olarak bulunur.

Euler Diferansiyel Denklemi

x^2y''-4xy'+6y=lnx diferansiyel denklemini çözünüz.

(x^2y''+Axy'+By=f(x) tipi euler dif denklem)

$$y = x^r$$
$$y' = rx^{r-1}$$
$$y'' = r(r-1)x^{r-2}$$

yazılarak $(r(r-1) - 4r + 6)x^r = 0$

r^2-5r+6=0 karakteristik denklemden (r_1=2 , r_2=3)

2 farklı reel kök olduğundan homojen çözüm;

$$y_{\text{homojen}} = c_1 x^{r_1} + c_2 x^{r_2} = c_1 x^2 + c_2 x^3$$

İkinci taraflı denklemin çözümü için Wc'= ε_n sistemini yazarsak

W= $\begin{bmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{bmatrix}$ $c' = \begin{bmatrix} c_1' \\ c_2' \end{bmatrix}$ $\varepsilon_n = \begin{bmatrix} 0 \\ \dfrac{f(x)}{x^n} \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{\ln x}{x^2} \end{bmatrix}$

$c_1' x^2 + c_2' x^3 = 0$

$c_1' 2x + 3c_2' x^2 = \dfrac{\ln x}{x^2}$

$-2c_1' x^2 - 2c_2' x^3 = 0$

$c_1' 2x^2 + 3c_2' x^3 = \dfrac{\ln x}{x}$

$c_2' = \dfrac{\ln x}{x^4}$

u=lnx dv=x^{-4}dx u v- $\int v du = -\dfrac{\ln x}{3x^3} - \int -\dfrac{1}{3x^4} dx = -\dfrac{\ln x}{3x^3} - \dfrac{1}{9x^3} + K_2$ =c$_2$

du=(1/x)dx v=- $\dfrac{1}{3x^3}$

$c_1' = -\dfrac{\ln x}{x^3}$ c$_1$= $-\dfrac{\ln x}{2x^2} - \dfrac{1}{x^2} + K_1$

y$_{\text{genel}}$= $K_1 x^2 + K_2 x^3 - \dfrac{5 \ln x}{6} - \dfrac{10}{9}$

LAPLACE DÖNÜŞÜMÜ

1)

$f(t) = t^3 \cos t = t^2(t \cos t)$

$t^n g(t)$, for $n = 1, 2, \ldots$	$(-1)^n \frac{d^n G(s)}{ds^n}$

$$G(s) = \mathcal{L}\{t \cos t\} = \frac{s^2 - 1}{(s^2+1)^2}$$

$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2+\omega^2)^2}$

$\mathcal{L}\{t f(t)\} = -F'(s) = -\frac{d}{ds}F(s)$ den

$G(s)$ nin 2. Türevine ihtiyacımız olduğundan

1. türev

$$\frac{d}{ds}\frac{s^2-1}{(s^2+1)^2} = -2s\frac{s^2-3}{(s^2+1)^3}$$

2.türev

$$\frac{d^2}{ds^2}\frac{s^2-1}{(s^2+1)^2} = 6\frac{s^4-6s^2+1}{(s^2+1)^4}$$

$(-1)^2 = 1$.

Sonuç olarak

$$\mathcal{L}\{t^3 \cos t\} = 6\frac{s^4-6s^2+1}{(s^2+1)^4}$$

2)

$f(t) = \cos^2 3t$, $\qquad \mathcal{L}\{\cos^2 t\} = \dfrac{s^2 + 2}{s(s^2 + 4)}$

$$\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

Burada, $a = 3$

$$\mathcal{L}\{\cos^2 3t\} = \frac{1}{3} \frac{\left(\frac{s}{3}\right)^2 + 2}{\left(\frac{s}{3}\right)\left(\left(\frac{s}{3}\right)^2 + 4\right)}$$

$$= \frac{\frac{s^2}{9} + 2}{s\left(\frac{s^2}{9} + 4\right)}$$

$$= \frac{s^2 + 18}{s(s^2 + 36)}$$

3) $G(s) = \dfrac{2}{s}(e^{-3s} - e^{-4s})$ ters laplace dönüşümünü bulunuz.

tablodan

$$\mathcal{L}\left\{\frac{e^{-as}}{s}\right\} = u(t-a)$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s}(e^{-3s} - e^{-4s})\right\}$$

$$= 2\left[\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s}\right\}\right]$$

$$= 2[u(t-3) - u(t-4)]$$

$g(t) = 2(u(t-3) - u(t-4))$

4)

$$G(s) = \frac{2s+1}{s^2}e^{-2s} - \frac{3s+1}{s^2}e^{-3s}$$ ters laplace dönüşümünü bulunuz

tablodan

$$\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2}e^{-2s} - \frac{3s+1}{s^2}e^{-3s}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2}{s}e^{-2s} + \frac{1}{s^2}e^{-2s} - \frac{3}{s}e^{-3s} - \frac{1}{s^2}e^{-3s}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}e^{-2s}\right\}$$ için

$$\mathcal{L}^{-1}\left\{e^{-2s} \times \frac{1}{s^2}\right\}$$

Şeklinde yazılarak $\mathcal{L}^{-1}\{e^{-as}G(s)\} = u(t-a) \cdot g(t-a)$ den

$$g(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$$

$$\mathcal{L}^{-1}\left\{e^{-2s} \times \frac{1}{s^2}\right\} = u(t-2) \times (t-2) = (t-2) \cdot u(t-2)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}e^{-3s}\right\} = \mathcal{L}^{-1}\left\{e^{-3s} \times \frac{1}{s^2}\right\} = (t-3) \cdot u(t-3)$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s}e^{-2s} + \frac{1}{s^2}e^{-2s} - \frac{3}{s}e^{-3s} - \frac{1}{s^2}e^{-3s}\right\}$$

$$= 2u(t-2) + (t-2) \cdot u(t-2) - 3u(t-3) - (t-3) \cdot u(t-3)$$

$$= 2u(t-2) + t \cdot u(t-2) - 2 \cdot u(t-2) - 3 \cdot u(t-3) - t \cdot u(t-3) + 3 \cdot u(t-3)$$

$$= t \cdot [u(t-2) - u(t-3)]$$

$g(t) = t \cdot (u(t-2) - u(t-3))$ olarak bulunur.

5- h(t)= $\begin{cases} 1 & 3 \le t < 12 \\ 0 & 0 \le t < 3 \ \ ve \ t \ge 12 \end{cases}$

olmak üzere

y''+y'+y=h(t), y(0)=0, y'(0)=0 Y(S) fonksiyonunu bulunuz.

Basamak fonksiyonunun tanımından

$U_3(t)$= $\begin{cases} 0 & t < 3 \\ 1 & t \ge 3 \end{cases}$ ***$U_{12}(t)$=*** $\begin{cases} 0 & t < 12 \\ 1 & t \ge 12 \end{cases}$ **=** $\begin{cases} 0 & t < 3 \\ 0 & 3 \le t < 12 \\ 1 & t \ge 12 \end{cases}$

Bu iki fonksiyonun farkı

H(t)= ***$U_3(t)$- $U_{12}(t)$=*** $\begin{cases} 1 & 3 \le t < 12 \\ 0 & 0 \le t < 3 \ \ ve \ t \ge 12 \end{cases}$

Şeklinde yazılabildiğinden başlangıç değer problemi

y''+y'+y= ***$U_3(t)$- $U_{12}(t)$*** **y(0)=0, y'(0)=0 şeklinde yazılabilir.**

Laplace dönüşümünün lineerliğinden

L[y'']+L[y']+L[y]= L[U_3(t)]-L[U_{12}(t)] ***yazılarak***

$$L[y''] = s^2Y(s) - sy(0) - y'(0)$$
$$L[y'] = sY(s) - y(0)$$
$$L[y] = Y(s)$$

$$s^2Y(s) - sy(0) - y'(0) + sY(s) - y(0) + Y(s) = \frac{e^{-3s}}{s} - \frac{e^{-12s}}{s}$$

$$s^2Y(s) + sY(s) + Y(s) = \frac{e^{-3s}}{s} - \frac{e^{-12s}}{s}$$

$$Y(s)(s^2 + s + 1) = \frac{e^{-3s}}{s} - \frac{e^{-12s}}{s}$$

$$Y(s) = \frac{1}{s(s^2 + s + 1)}(e^{-3s} - e^{-12s})$$

Çözümü:

veya

$$Y(s) = G(s)(e^{-3s} - e^{-12s})$$

olur.

y= L^{-1}(Y(s))= L^{-1}{e^{-3s}H(s)}- L^{-1}{ e^{-12s}H(s)}

$L^{-1}\left\{e^{-cs}F(s)\right\} = u_c(t)\,f(t-c)$ dikkate alınarak

y= u_3(t)f(t-3)-u_{12}(t)f(t-12) şeklinde bulunur.

f(t) fonksiyonu kapalı olarak verildiğinden basit kesirlere ayrılarak bulunur..

$$H(s) = \frac{1}{s(s^2+s+1)} = \frac{a}{s} + \frac{bs+c}{s^2+s+1}$$ *a=1, b=-1 ve c=-1*

$H(s) = \frac{1}{s} + \frac{-s-1}{s^2+s+1}$ **veya** $H(s) = \frac{1}{s} - \frac{s}{(s+\frac{1}{2})^2+\frac{3}{4}} - \frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}}$

$e^{at}\sin bt = \frac{b}{(s-a)^2+b^2}$ $\qquad$ $e^{at}\cos bt = \frac{(s-a)}{(s-a)^2+b^2}$

h(t) = $\frac{1}{s} - \frac{s}{(s+\frac{1}{2})^2+(\sqrt{3/4})^2} - \frac{\sqrt{3/4}\sqrt{4/3}}{(s+\frac{1}{2})^2+(\sqrt{3/4})^2}$ = 1-$e^{-1/2t}$cos $\sqrt{3/4}$ t- ($\sqrt{4/3}$ $e^{-1/2t}$sin $\sqrt{3/4}$ t)*-1/2

$L^{-1}\left\{e^{-cs}H(s)\right\} = u_c(t)h(t-c)$ dikkate alınarak

y(t)= u_3(t)h(t-3)-u_{12}(t)h(t-12) şeklinde bulunur.

Y(t)= 1-$e^{-1/2(t-3)}$cos $\sqrt{3/4}$ (t-3)- ($\sqrt{4/3}$ $e^{-1/2(t-3)}$sin $\sqrt{3/4}$ (t-3))*-1/2

6) $y''-2y'+y=\cos t$ $y(0)=0,$ $y'(0)=1$

$L\{y''\}-2L\{y'\}+L\{y\}=L\{\cos t\}$

$L[y'']=s^2Y(s)-sy(0)-y'(0)$
$L[y']=sY(s)-y(0)$
$L[y]=Y(s)$

$s^2Y(s)-1-2sY(s)+Y(s)=\frac{s}{s^2+1}$

$Y(s)\left[s^2-2s+1\right]=\frac{s}{s^2+1}$

$$Y(s)=\frac{s}{(s-1)^2(s^2+1)}$$

Basit kesirlere ayırarak

$$\frac{s}{(s-1)^2(s^2+1)}=\frac{a}{s-1}+\frac{b}{(s-1)^2}+\frac{cs+d}{(s^2+1)}$$

$a(s^2+1)(s-1)+b(s^2+1)+(cs+d)(s^2-2s+1)=s$ $as^3-as^2+as-a+bs^2+b+cs^3-2cs^2+cs+ds^2-2ds+d=2$

$s^3(a+c)=0$
$s^2(-a+b-2c+d)=0$
$s(a+c-2d)=s$
$-a+b+d=0$
$a+c=0$ $a=-c$ $-a+b-2c+d=0$ $-a+b+2a+d=0$ $a+b+d=0$
$a+c-2d=1$ $-a+b+d=0$ $-a+b+d=0$

d=-1/2 b=1/2 a=0 c=0

$\frac{s}{(s-1)^2(s^2+1)}=\frac{1/2}{(s-1)^2}+\frac{-1/2}{(s^2+1)}$ $=L^{-1}(\frac{1/2}{(s-1)^2})+L^{-1}(\frac{-1/2}{(s^2+1)})$ **=1/2te^t+-1/2sint**

$L^{-1}\left(\frac{n!}{(s-a)^{n+1}}\right)=t^ne^{at}$ $L^{-1}\left(\frac{a}{s^2+a^2}\right)=\sin at$

7) $y''-2y'+y=\sin t$ $\quad\quad y(0)=0$**,** $\quad\quad y'(0)=1$

$L\{y''\}-2L\{y'\}+L\{y\}=L\{\sin t\}$

$L\{y''\}=s^2Y(s)-sy(0)-y'(0)$
$L\{y'\}=sY(s)-y(0)$
$L\{y\}=Y(s)$

$s^2Y(s)-1-2sY(s)+Y(s)=\dfrac{1}{s^2+1}$

$Y(s)\left[s^2-2s+1\right]=\dfrac{1}{s^2+1}$

$$Y(s)=\frac{1}{(s-1)^2(s^2+1)}$$

Basit kesirlere ayırarak

$$\frac{1}{(s-1)^2(s^2+1)}=\frac{a}{s-1}+\frac{b}{(s-1)^2}+\frac{cs+d}{(s^2+1)}$$

$a(s^2+1)(s-1)+b(s^2+1)+(cs+d)(s^2-2s+1)=s$ $\quad as^3-as^2+as-a+bs^2+b+cs^3-2cs^2+cs+ds^2-2ds+d=2$

$s^3(a+c)=0$
$s^2(-a+b-2c+d)=0$
$s(a+c-2d)=0$
$-a+b+d=1$

$a+c=0$ $\quad a=-c$ $\quad\quad -a+b-2c+d=0$ $\quad -a+b+2a+d=0$ $\quad a+b+d=0$
$a+c-2d=0$ $\quad\quad -a+b+d=1$ $\quad\quad -a+b+d=1$

$d=0$ $\quad b=1/2$ $\quad a=-1/2$ $\quad c=1/2$

$$\frac{1}{(s-1)^2(s^2+1)}=\frac{-1/2}{s-1}+\frac{1/2}{(s-1)^2}+\frac{1/2s}{(s^2+1)}=L^{-1}(\frac{-1/2}{s-1})+L^{-1}(\frac{1/2}{(s-1)^2})+L^{-1}(\frac{1/2s}{(s^2+1)})$$

y(t)= $-1/2e^t+1/2te^t+1/2\cos t$

8) $y'' + 2y' + 5y = \sin(3t), \quad y(0) = 1 \quad y'(0) = -1$ **denklemini çözünüz.**

$$L(y'') + 2L(y') + 5L(y) = L\Big(\sin(3t)\Big) \ .$$

$$L(y'') = s^2 L(y) - sy(0) - y'(0) = s^2 L(y) - s + 1,$$

$$L(y') = sL(y) - y(0) = sL(y) - 1 \ ,$$

$$s^2 L(y) - s + 1 + 2\Big(sL(y) - 1\Big) + 5L(y) = \frac{3}{s^2 + 9} \ .$$

$$(s^2 + 2s + 5)L(y) = s + 1 + \frac{3}{s^2 + 9},$$

$$L(y) = \frac{s + 1 + \dfrac{3}{s^2 + 9}}{(s^2 + 2s + 5)} = \frac{s + 1}{(s^2 + 2s + 5)} + \frac{3}{(s^2 + 9)(s^2 + 2s + 5)} \ .$$

Ters laplace dönüşümü için tablodan

$$\frac{s + 1}{(s^2 + 2s + 5)} = \frac{s + 1}{(s + 1)^2 + 4} = L\Big(e^{-t}\cos(2t)\Big) \ .$$

2. terim için kesirlere ayırma yöntemi ile

$$A = -\frac{3}{26} \ , \ \ B = -\frac{3}{13} \ , \ \ C = \frac{3}{26} \ , \ \ D = -\frac{6}{13} \ .$$

$$\frac{3}{(s^2 + 9)(s^2 + 2s + 5)} = \frac{3}{26}\left(-\frac{s + 2}{s^2 + 9} + \frac{s + 4}{s^2 + 2s + 5}\right) \ .$$

tablodan

$$\frac{s + 2}{s^2 + 9} = \frac{s}{s^2 + 9} + \frac{2}{s^2 + 9} = L\left(\cos(3t) + \frac{2}{3}\sin(3t)\right)$$

tablodan

$$\frac{s + 4}{s^2 + 2s + 5} = \frac{s + 1}{(s + 1)^2 + 4} + \frac{3}{(s + 1)^2 + 4} = \mathcal{L}\left(e^{-t}\cos(2t) + \frac{3}{2}e^{-t}\sin(2t)\right) \ .$$

Sonuç olarak

$$y(t) = e^{-t}\cos(2t) + \frac{3}{26}\left(-\cos(3t) - \frac{2}{3}\sin(3t) + e^{-t}\cos(2t) + \frac{3}{2}e^{-t}\sin(2t)\right) \ .$$

9) $y''-3y'+2y=\delta(t)+u_2(t)$ $y(0)=0$, $y'(0)=0$ başlangıç değer problemini çözünüz.

Laplace dönüşümünün lineerliliğinden

$L(y'')-3L(y')+2L(y)=L(\delta(t))+L(u_2(t))$

$L(y'')=s^2Y(s)-s\,y(0)-y'(0)$ $L(\delta(t))=1$ $L(u_2(t))=e^{-2s}/s$

$L(y') = sY(s)-y(0)$

$L(y) = Y(s)$

$$Y(s)=\frac{1}{(s-1)(s-2)}+\frac{e^{-2s}}{s(s-1)(s-2)}$$

$F(s)=\dfrac{1}{s(s-1)(s-2)}$ basit kesirlere ayırma yöntemi ile;

$$\frac{1}{s(s-1)(s-2)}=\frac{a}{s}+\frac{b}{s-1}+\frac{c}{s-2}$$ a=c=1/2, b=-1

$$\frac{1}{(s-1)(s-2)}=\frac{d}{s-1}+\frac{e}{s-2}$$ d=-1, e=1

$$Y(s)=\frac{-1}{s-1}+\frac{1}{s-2}+\left(\frac{1/2}{s}+\frac{-1}{s-1}+\frac{1/2}{s-2}\right)e^{-2s}$$

$$L^{-1}(Y(s))=L^{-1}\left(\frac{-1}{s-1}\right)+L^{-1}\left(\frac{1}{s-2}\right)+\frac{1}{2}\left(L^{-1}\left(\frac{1}{s}\right)-L^{-1}\left(\frac{2}{s-1}\right)+L^{-1}\left(\frac{1}{s-2}\right)\right)e^{-2s}$$

$L^{-1}\left(\dfrac{1}{s-a}\right)=e^{at}$ $F(s)=\dfrac{1}{s(s-1)(s-2)}$ $L^{-1}(F(s))=1/2(1-2e^{t}+e^{2t})=f(t)$

$L(u_c(t)f(t-c))=e^{-cs}F(s)$; $L^{-1}(e^{-cs}F(s))=u_c(t)f(t-c)$ kullanılarak

$$\mathbf{y(t)=-e^{t}+e^{2t}+1/2(1-2e^{t-2}+e^{2(t-2)})u_2(t)}$$

Laplace dönüşümü tablosu

Fonksiyon $f(t)$ $f(t) = \mathcal{L}^{-1}\{F(s)\}$	**Laplace Dönüşümü $f(t)$** $F(s) = \mathcal{L}\{ f(t)\}$
1	$\frac{1}{s}$ $s > 0$
t (unit-ramp function)	$\frac{1}{s^2}$ $s > 0$
t^n (n, a positive integer)	$\frac{n!}{s^{n+1}}$ $s > 0$
e^{at}	$\frac{1}{s-a}$ $s > a$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$ $s > 0$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$ $s > 0$
$t^n g(t)$, for $n = 1, 2, ...$	$(-1)^n \frac{d^n G(s)}{ds^n}$
$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$ $s > \lvert\omega\rvert$
$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$ $s > \lvert\omega\rvert$
$g(at)$	$\frac{1}{a} G(\frac{s}{a})$ Scale property
$e^{at} g(t)$	$G(s - a)$ Shift property
$e^{at} t^n$, for $n = 1, 2, ...$	$\frac{n!}{(s-a)^{n+1}}$ $s > a$
te^{-t}	$\frac{1}{(s+1)^2}$ $s > -1$
$1 - e^{-t/T}$	$\frac{1}{s(1+Ts)}$ $s > -1/T$

$e^{at}\sin \omega t$	$\dfrac{\omega}{(s-a)^2+\omega^2}$ $s > a$
$e^{at}\cos \omega t$	$\dfrac{s-a}{(s-a)^2+\omega^2}$ $s > a$
$u(t)$	$\dfrac{1}{s}$ $s > 0$
$\delta(t-a)$	$\dfrac{e^{-as}}{s}$ $s > 0$
$u(t-a)g(t-a)$	$e^{-as}G(s)$ Time-displacement theorem
$g'(t)$	$sG(s) - g(0)$
$g''(t)$	$s^2 \bullet G(s) - s \bullet g(0) - g'(0)$
$g^{(n)}(t)$	$s^n \bullet G(s) - s^{n-1} \bullet g(0) - s^{n-2} \bullet g'(0) - \dots - g^{(n-1)}(0)$
$\int_0^t g(t)dt$	$\dfrac{G(s)}{s}$
$\int g(t)dt$	$\dfrac{G(s)}{s} + \dfrac{1}{s}\left\{\int g(t)dt\right\}_{t=0}$

10) $Y(s) = \dfrac{1}{(s^2-3s-4)}$ **konvolüsyon teoremini kullanarak ters laplace dönüşümünü bulunuz**

$Y(s) = \dfrac{1}{(s^2-3s-4)} = \dfrac{1}{(s-4)(s+1)}$ **ters laplace dönüşümünün lineerliliğinden**

$$L^{-1}\left(\frac{1}{(s-4)(s+1)}\right) = L^{-1}\left(\frac{1}{s-4}\right) + L^{-1}\left(\frac{1}{s+1}\right)$$

$L^{-1}\left(\dfrac{1}{s-a}\right) = e^{at}; \quad L^{-1}\left(\dfrac{1}{s+a}\right) = e^{-at}$ yararlanarak

$L^{-1}\left(\dfrac{1}{s-4}\right) = e^{4t} = f(t)$, $L^{-1}\left(\dfrac{1}{s+1}\right) = e^{-t} = g(t)$ *ise*

$$\int_0^t f(\tau)g(t-\tau)d\tau = \int_0^t e^{4\tau}e^{-(t-\tau)}d\tau = \frac{1}{5}e^{(5\tau-t)}\Big|_0^t = \frac{1}{5}e^{4t} - \frac{1}{5}e^{-t}$$

y''+2y'+y=0 2. mertbeden homojen lineer diferansiyel denkleme karşı gelen 1. merftebeden diferansiyel denklem sistemini bulunuz. Bulduğunuz 1. mertebeden diferansiyel denklem sistemini x'=Ax formunda yazınız.

Çözüm:

x_1=y ve x_2=y' dönüşümü kullanılarsa ve $x' = y' = x_2$ ve $x_2' = y''$ ile

$$\left.\begin{aligned} y &= x_1 \\ y' &= x_2 \\ y'' &= x_2' \end{aligned}\right\} \quad y''+2y'+y=0 \, yerlerine konursa$$

$$x_2' + 2x_2 + x_1 = 0$$

bulunur. Dolayısıyla x_1 ve x_2 aşağıdaki 1.mertebeden diferansiyel denklemi sağlar.

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_1 - 2x_2 \end{aligned}$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad x' = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} x \qquad x'=Ax$$

Örnek

$$x' = \begin{pmatrix} 1 & 9 \\ -1 & -5 \end{pmatrix} x, \qquad x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ **başlangıç değer problemini çözünüz.**

$$\begin{bmatrix} (1-\lambda) & 9 \\ -1 & (-5-\lambda) \end{bmatrix} = \lambda^2 + 4\lambda + 4 = (\lambda+2)^2$$ $\lambda_1=\lambda_2=-2$ katlı kök

$\lambda_1=-2$ katlı kök için özvektör

$$\vec{x}_1 = \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\vec{x}_2 = \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathbf{x_{genel}} = c_1 \vec{x}_1 e^{\lambda t} + c_2 (\vec{x}_1 t + \vec{x}_2) e^{\lambda t}$$

$$\mathbf{x_{genel}} = c_1 \begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{-2t} + c_2 \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) e^{-2t}$$

Başlangıç koşuları dikkate alınarak t=0 ile

$x(0) = c_1 \begin{pmatrix} 3 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $c_1=1$, $c_2=-2$ bulunur.

$$\mathbf{x_{genel}} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{-2t} - 2 \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) e^{-2t}$$